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New equations of infinitesimal deformations of surfaces in E_3. (English summary)

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For a surface in three-dimensional Euclidean space, given by the parametric equation $\overline{r} = \overline{r}(x^1, x^2)$, the author studies an infinitesimal deformation defined by

$$\overline{r}_t = \overline{r}(x^1, x^2) + t\overline{y}(x^1, x^2)$$

where $\overline{y}(x^1, x^2)$ is a displacement vector.

On the one hand it is proven that for such an infinitesimal deformation there exist two symmetric tensor fields α_{ij} and β_{ij} that satisfy the relations

$$\beta_{ik}b_{jl} - \beta_{ij}b_{kl} + \beta_{jl}b_{ik} - \beta_{kl}b_{ij} = g_{ml}\delta R^m_{ijk} + \alpha_{ml}R^m_{ijk} \tag{1}$$

and

$$\beta_{ij,k} - \beta_{ik,j} = b_{mj}\delta\Gamma^m_{ik} - b_{mk}\delta\Gamma^m_{ij} \tag{2}$$

where the tensor fields $\delta\Gamma^h_{ij}$ and δR^h_{ijk} are the variations of the Christoffel symbols of the second kind and of the Riemann tensor, "," is the covariant derivative based on the metric tensor g_{ij} of the surface and R^h_{ijk} are the components of the Riemann curvature tensor.

On the other hand the author shows that if, for a surface, there exist α_{ij} and β_{ij} that satisfy relations (1) and (2), then there exists an infinitesimal deformation of the surface for which these tensor fields are the variations δg_{ij} and δb_{ij} of the coefficients of the first and the second fundamental forms of the surface, respectively.

The analogy with the Gauss equations and Mainardi-Peterson-Codazzi equations in the classical surface theory is pointed out.

Reviewed by *Wendy Goemans*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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